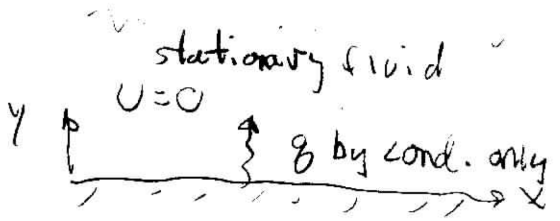
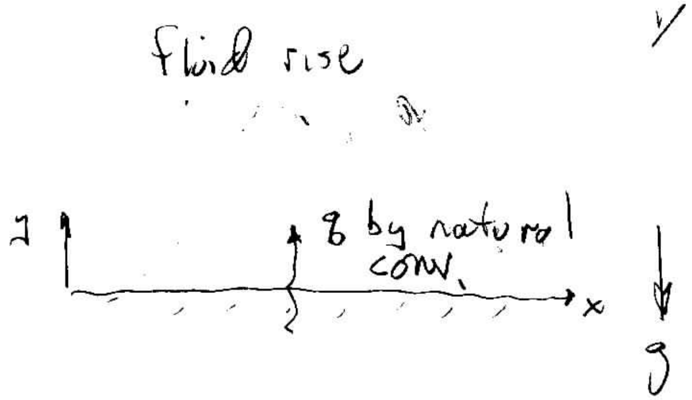
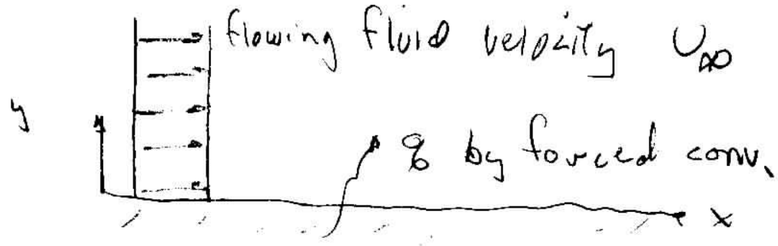
Convection





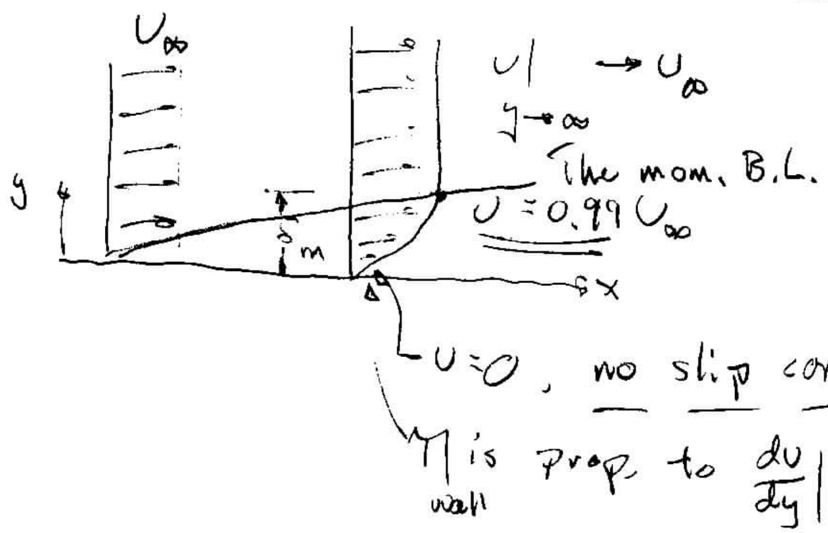


- Fluid viscosity (absolute) (Dynamic) μ $\left[\frac{kg}{ms} \sim \frac{Ns}{m^2} \sim Pa\,s, \quad 1 Pa\,s \pm 1 poise\right]$

  Kinematic $\nu = \frac{\mu}{\rho}$ $\left[\frac{m^2}{s} \sim \text{stoke}, \; 1\,\text{stoke} = 1\frac{cm^2}{s} = 0{,}0001\, m^2/s\right]$

- Wall shear stress $\tau_{wall} \equiv \mu \frac{\partial u}{\partial y}\Big|_{y=0}$ $[N/m^2]$

  (Newtonian fluids ~ linear relation)

- 

note $u = u(y)$ only

$v = 0$

$w = 0$

$\underline{V} = u\hat{i} + v\hat{j} + w\hat{k}$

But you need to know $u(y)$ to calc. τ_{wall}.

or

Use a total applied shear stress over the entire surface at any location

$$\tau_w = \frac{C_f}{2} \rho V^2 \quad [N/m^2]$$

C_f ~ dimensionless friction coef or skin coef

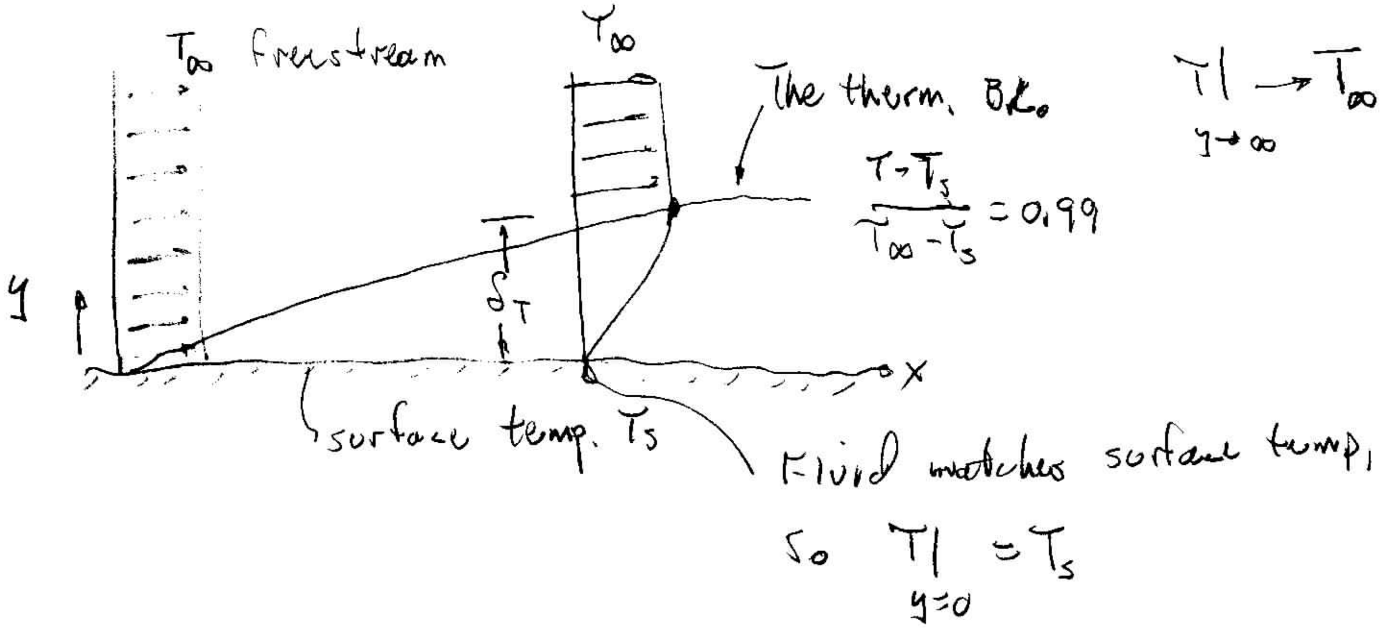
V ~ upstream velocity (far to left)

Note that C_f varies $C_f(x)$. So can define average total force over entire surface A_s

$$F_{friction} = \frac{1}{2} C_f A_s \rho V^2 \quad [N]$$

Ultimately F_f will be related to a heat transfer coef.

Thermal Boundary Layer

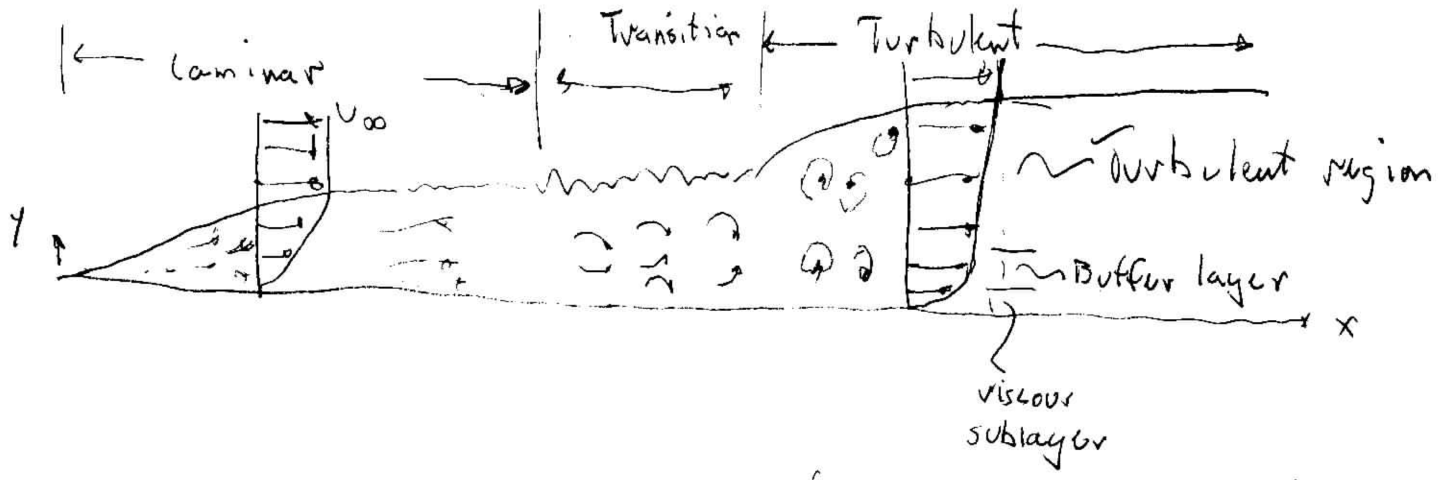


• Prandtl #

$$Pr \sim \frac{\text{mom. diffusion}}{\text{therm. diffusion}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k} \quad \text{dimensionless}$$

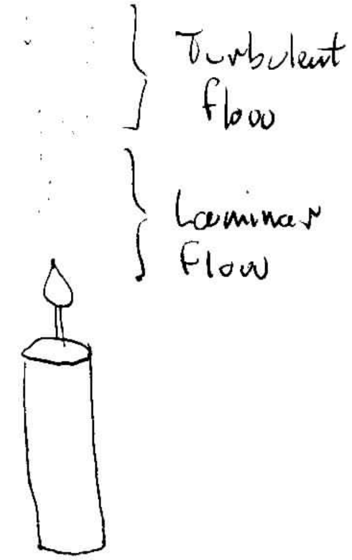
Liq metals — gases, 2~1 — Water ~ 1~10 — oils

small ~ 10^{-3} ← Pr → large 10^3 ~ 10^4



$$\left|\frac{du}{dy}\right|_{\text{viscous layer}} < \left|\frac{du}{dy}\right|_{\text{Turb. region}}$$

Candle smoke



So what Determines when laminar flow → turb. flow

Look at

$$Re \equiv \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_\infty L_{char}}{\nu} = \frac{\rho V_\infty L_{char}}{\mu}$$

dimensionless $\quad [m^2/s]$

$\sim \dfrac{\text{momentum dif.}}{\text{viscous dif}}$

$\sim \dfrac{\text{overturning}}{\text{damping}}$

Flat plate $Re_{crit} \sim 5\times10$

Pipe flow $\sim 2\times10^3$

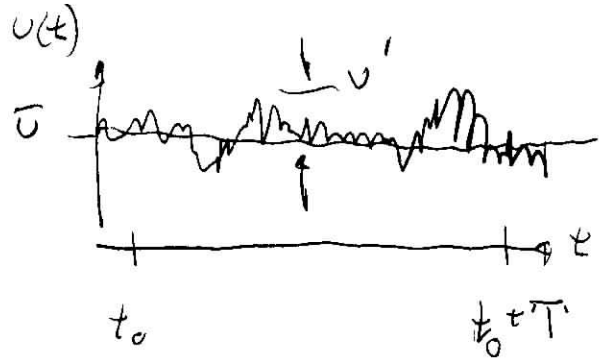
~ varies with geometries + flow conditions

Average prop. value $\bar{u}$

Inst. prop. value u $\qquad u = \bar{u} + u'$ (inst., avg, transient or flux. from $\bar{u}$)

flux. prop. value u'

Average value of a property $\qquad \bar{u} \equiv \frac{1}{T}\int_{t=t_0}^{t_0+T} u(t)\,dt$



Lots o' calculations + definitions

... $\quad \tau_{turbulent} = -\rho\overline{u'v'} = \mu_t \frac{\partial \bar{u}}{\partial y} \qquad \dot{q}_{turb} = \rho c_p \overline{v'T'} = -k_{turb}\frac{\partial \bar{T}}{\partial y}$

and so on.

lots of "second order" terms resulting from the ( )' values

Ex) μ_{turb} ~ turbulent or eddy viscosity

So overall $\qquad \tau_{total} = (\mu + \mu_t)\frac{\partial \bar{u}}{\partial y} = \rho(\nu + \nu_t)\frac{\partial \bar{u}}{\partial y}$ ←

$$q_{total} = -(k + k_T)\frac{\partial \bar{T}}{\partial y} = -\rho c_p(\alpha + \alpha_T)\frac{\partial \bar{T}}{\partial y}$$ ←

Conservation Equations (Mass, Mom, Thermal)

$\underline{V} = u(x,y)\hat{i} + v(x,y)\hat{j} + w(x,y)\hat{k}$, with $w = 0$

Mass $\quad \nabla \cdot \underline{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (\rho = c)$

x-Dir Mom. $\quad \rho \frac{Du}{Dt} = \rho \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} \quad$ (const. ρ, μ)

Plus 2 more for y & z momentum comp.

$P = P(x)$ only for no grav. or other body forces

Thermal Energy $\quad \rho C_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \mu \Phi$

Φ = viscous dissipation

$$= 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 \right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2$$

Hmm...

If viscous shear forces are negl. $\Phi \sim 0$ compared to

$$\rho C_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

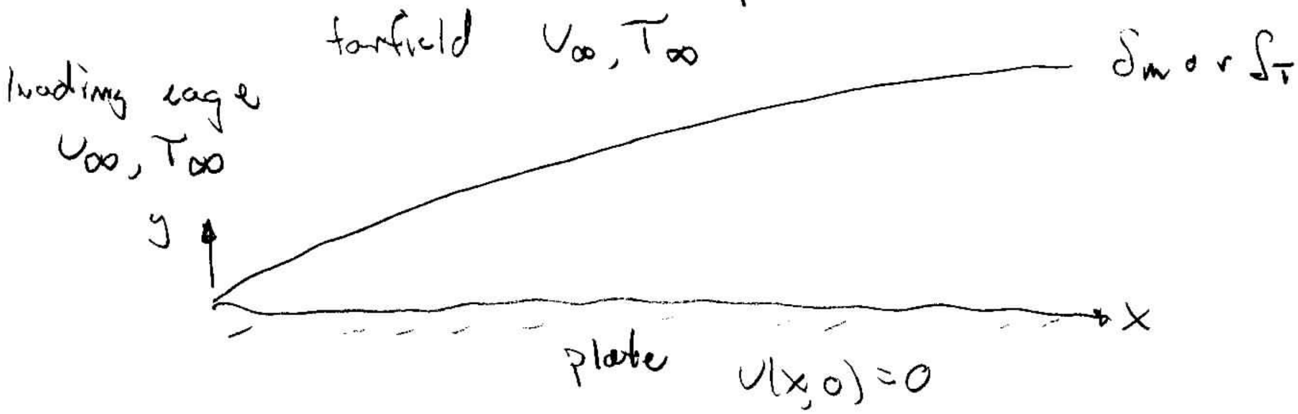
→ or $\quad \rho C_p \frac{DT}{Dt} = k \nabla^2 T \qquad T = T(x,y)$

also

→ $\quad \rho \frac{Du}{Dt} = \mu \nabla^2 u \qquad u = u(y)$ only

→ $\quad \nabla \cdot \underline{V} = 0 \qquad \underline{V} = u(y)\hat{i} + v(x,y)\hat{j} + 0\hat{k}$

Whole picture for a flat plate



$$u(x,0)=0$$
$$v(x,0)=0$$
$$T(x,0)=T_s$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}$$

B.C.

① $x=0$ leading edge of plate $u(0,y)=U_\infty$ $T(0,y)=T_\infty$

② $y=0$ at plate $u(x,0)=0$ $v(x,0)=0$ $T(x,0)=T_\infty$

③ $y\to\infty$ far field $u(x,\infty)=U_\infty$ $T(x,\infty)=T_\infty$

Note: that we never say $v(x,\infty)=0$!!!

Why?

We want $u=u(x,y;\ L, U_\infty, \rho, \mu,$

$T=T(x,y;\ L, U_\infty, \rho, \mu, T_s, T_\infty, k)$

Non-dimensionalize

- characteristic length L_c or L (plate length)
- characteristic velocity U_∞

$$\tilde{x} = \frac{x}{L} \quad \tilde{y} = \frac{y}{L} \quad \tilde{u} = \frac{u}{U_\infty} \quad \tilde{v} = \frac{v}{U_\infty} \quad \tilde{p} = \frac{P}{\rho U_\infty^2} \quad \Theta = \frac{T(x,y) - T_s}{T_\infty - T_s}$$

Reduce to

Mass
$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0$$

x-Mom.
$$\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = \frac{1}{Re_L}\frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} - \frac{d\tilde{p}}{d\tilde{x}}$$

$$Re_L = \frac{\rho U_\infty L}{\mu}$$

Thermal
$$\tilde{u}\frac{\partial \tilde{T}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{T}}{\partial \tilde{y}} = \frac{1}{Re_L Pr}\frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2}$$

$$Pr = \nu / \alpha$$

B.C.

$\tilde{u}(0,\tilde{y}) = 1$ $\quad$ $\tilde{v}(\tilde{x},0) = 0$ $\quad$ $\tilde{T}(0,\tilde{y}) = 1$

$\tilde{u}(\tilde{x},0) = 0$ $\quad$ $\tilde{T}(\tilde{x},0) = 0$

$\tilde{u}(\tilde{x},\infty) = 1$ $\quad$ $\tilde{T}(\tilde{x},\infty) = 1$

We are looking for $\tilde{u} = \tilde{u}(\tilde{x}, \tilde{y}; Re_L)$

$\tilde{T} = \tilde{T}(\tilde{x}, \tilde{y}; Re_L, Pr)$

$\tilde{v}$ is an afterthought

At the end of the day we want things like

$$\tau_{wall} = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \ldots = \left(\frac{\mu U_\infty}{L}\right) \frac{\partial \tilde{u}}{\partial \tilde{y}}\bigg|_{\tilde{y}=0} = \left(\frac{\mu U_\infty}{L}\right) f_1(\tilde{x}, Re_L)$$

$$C_{f,x} = \frac{\tau_{wall}}{\left(\frac{1}{2}\rho U_\infty^2\right)} = \ldots = \frac{(\mu U_\infty / L)}{\left(\frac{1}{2}\rho U_\infty^2\right)} \frac{\partial \tilde{u}}{\partial \tilde{y}}\bigg|_{\tilde{y}=0} = \frac{2}{Re_L} f_1(\tilde{x}, Re_L)$$

$$h_x = \frac{-k \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)} = \ldots = \frac{k}{L} \frac{\partial \tilde{T}}{\partial \tilde{y}}\bigg|_{\tilde{y}=0}$$

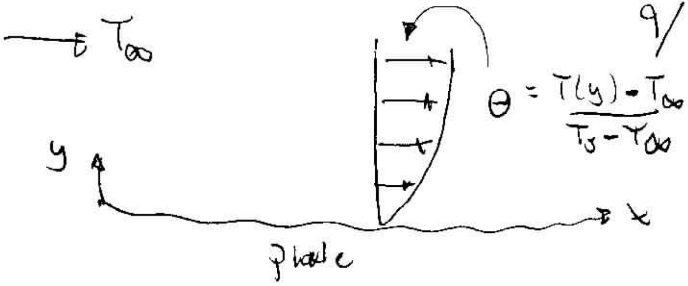
$$Nu_x = \frac{h_x L}{k} = \frac{\partial \tilde{T}}{\partial \tilde{y}}\bigg|_{\tilde{y}=0} = f_2(\tilde{x}, Re_L, Pr)$$

Ex1

Air @ $T_\infty = 20°C$

flows over flat plate

@ $T_s = 160°C$.



The dim. less temp is $\theta(y) = \frac{T(y) - T_\infty}{T_s - T_\infty} = e^{-ay}$

where $a = 3200\ m^{-1}$.

Determine - heat flux at plate surface

- convection heat transfer coef.

Sol^n Assume radiation is negli.

Calc. k air at average film temp $\bar{T}_{air} = \frac{T_s + T_\infty}{2} = 90°C$

From tables $k_{fluid} = 0.03024\ \frac{W}{mK}$

$q''_{cond.}$ to surface from plate interior $=$ q''_{conv} away from plate by fluid

$$q''_{cond} = -k_f \left.\frac{\partial T}{\partial y}\right|_{y=0} = -k_f (T_s - T_\infty)\, a\, e^{-ay}\Big|_{y=0}$$

$$= (T_s - T_\infty)(-a k_f)$$

$$= -4.48 \times 10^5\ \frac{°C}{m}(k_f)$$

$$= 1.35 \times 10^4\ W/m^2$$

Finally $h = \dfrac{-k_{fluid}\left(\left.\dfrac{\partial T}{\partial y}\right|_{y=0}\right)}{(T_s - T_\infty)} = 96.8 \dfrac{W}{m^2 K}$

Note one could also use Newton's law of cooling

$$q'' = h(T_s - T_\infty)$$